Identifying Causal Effects in Experiments with Spillovers and Non-compliance

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Empirical Example with Potential for Indirect Treatment Effects

Crepon et al. (2013; QJE)

- Large-scale job-seeker assistance program in France.
- Randomized offers of intensive job placement services.

Displacement Effects of Labor Market Policies

"Job seekers who benefit from counseling may be more likely to get a job, but at the expense of other unemployed workers with whom they compete in the labor market. This may be particularly true in the short run, during which vacancies do not adjust: the unemployed who do not benefit from the program could be partially crowded out."

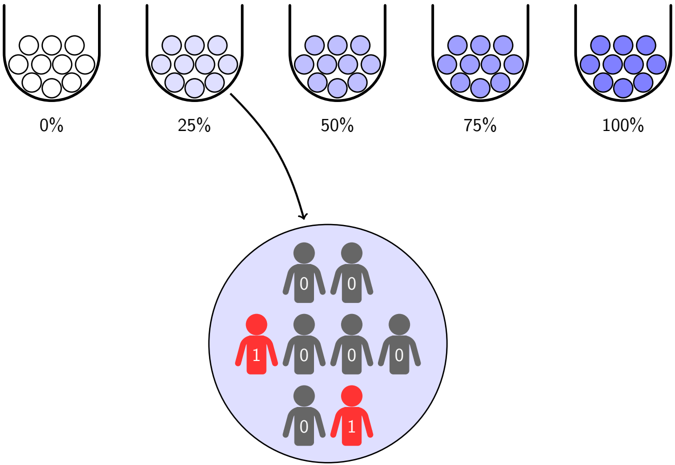
Studying Spillovers with Cluster Data

Partial Interference

Spillovers within but not between groups.

Randomized Saturation

Two-stage experimental design.



This Paper: Non-compliance in Randomized Saturation Experiments

Identification

Beyond Intent-to-Treat: Direct & indirect causal effects under 1-sided non-compliance.

Estimation

Simple, asymptotically normal estimator under large/many-group asymptotics.

Application

French labor market experiment: Crepon et al. (2013; QJE)

Notation

Sample Size and Indexing

Groups	$g = 1, \ldots G$
Individuals in g	$i = 1, \ldots, N_g$

Observables

Y_{ig}	outcome of (i, g)
$Z_{ig} \in \{0, 1\}$	treatment offer to (i, g)
$D_{ig} \in \{0, 1\}$	treatment take-up of (i, g)
$\bar{D}_{ig} \in [0, 1]$	take-up of (i, g)'s "neighbors"
$S_g \in \mathcal{S} \subseteq [0, 1]$	saturation of group g

Overview of Assumptions

(i) Experimental Design: Randomized Saturation ✓

(ii) Potential Outcomes: Correlated Random Coefficients Model

(iii) Treatment Take-up: 1-sided Noncompliance & "Individualized Offer Response"

(iv) Exclusion Restriction for (Z_{ig}, S_g)

(v) Rank Condition

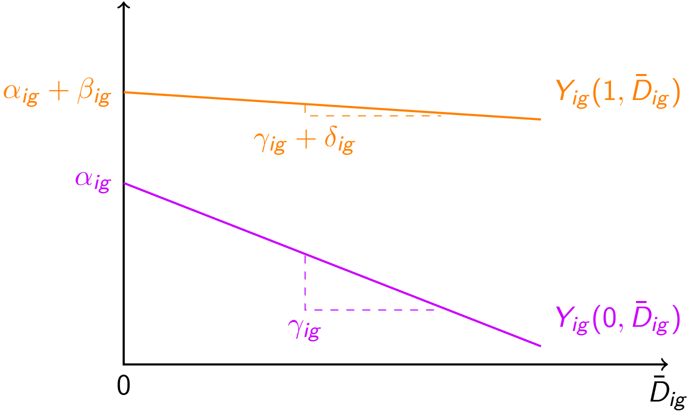
Assumption (ii) – Correlated Random Coefficients (CRC) Model

$$Y_{ig}(\boldsymbol{D}) = Y_{ig}(\boldsymbol{D}_g) = Y_{ig}(D_{ig}, \bar{D}_{ig}) = \mathbf{f}(\bar{D}_{ig})' \left[(1 - D_{ig})\boldsymbol{\theta}_{ig} + D_{ig}\boldsymbol{\psi}_{ig}\right]$$

$\mathbf{f}$	vector of known functions	Lipschitz continuous on $[0,1]$
$(\boldsymbol{\theta}_{ig}, \boldsymbol{\psi}_{ig})$	random variables	possibly dependent on $(D_{ig}, \bar{D}_{ig})$

This Talk – linear potential outcomes model...

$$Y_{ig}(D_{ig}, \bar{D}_{ig}) = \alpha_{ig} + \beta_{ig} D_{ig} + \gamma_{ig} \bar{D}_{ig} + \delta_{ig} D_{ig} \bar{D}_{ig}$$



Indirect Effects

Treated: $\gamma_{ig} + \delta_{ig}$

Untreated: γ_{ig}

Direct Effects

$\beta_{ig} + \delta_{ig} \bar{D}_{ig}$

Assumption (iii) – Treatment Take-up

1-sided Non-compliance

Only those offered treatment can take it up.

Individualistic Offer Response (IOR)

$$D_{ig}(\boldsymbol{Z}) = D_{ig}(\boldsymbol{Z}_g) = D_{ig}(Z_{ig})$$

Notation

$C_{ig} = 1$ iff (i, g) is a complier; $\bar{C}_{ig} \equiv$ share of compliers among (i, g)'s neighbors.

$$\boxed{\text{(IOR)} + \text{(1-Sided)} \Rightarrow D_{ig} = C_{ig} \times Z_{ig}}$$

No Evidence Against IOR in Our Example

Data from Crepon et al. (2013; QJE)

(IOR) + (1-Sided)

Take-up only depends on *own* offer:

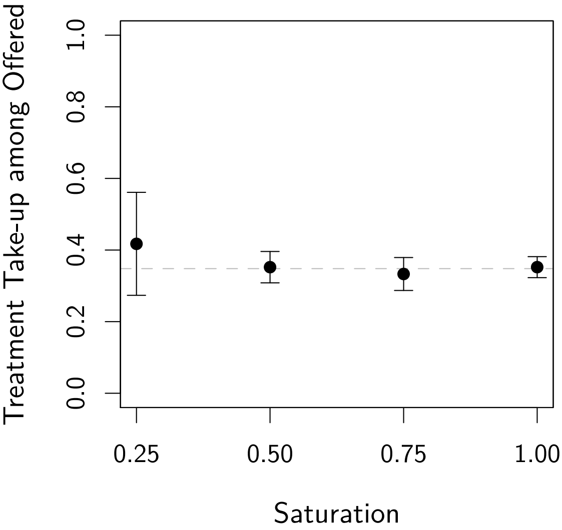
$D_{ig} = C_{ig} \times Z_{ig}$

Testable Implication

$\mathbb{E}[D_{ig}|Z_{ig} = 1, S_g] = \mathbb{E}[D_{ig}|Z_{ig} = 1]$

Figure at right

Take-up among offered doesn't vary with saturation (p = 0.62)



Assumption (iv) – Exclusion Restriction

Notation

$\mathbf{B}_g$	random coefficients for everyone in group g
$\boldsymbol{C}_g$	complier indicators for everyone in group g
$\boldsymbol{Z}_g$	treatment offers for everyone in group g

Exclusion Restriction

(i) $S_g \perp\!\!\!\perp (\boldsymbol{C}_g, \mathbf{B}_g, N_g)$

(ii) $\boldsymbol{Z}_g \perp\!\!\!\perp (\boldsymbol{C}_g, \mathbf{B}_g) | (S_g, N_g)$

Näive IV Does Not Identify the Spillover Effect

Unoffered Individuals

$$
\begin{aligned}
Y_{ig} &= \alpha_{ig} + \cancel{\beta_{ig} D_{ig}} + \gamma_{ig} \bar{D}_{ig} + \cancel{\delta_{ig} D_{ig} \bar{D}_{ig}} \\
&= \underbrace{\mathbb{E}[\alpha_{ig}]}_{\alpha} + \underbrace{\mathbb{E}[\gamma_{ig}]}_{\gamma} \bar{D}_{ig} + \underbrace{(\alpha_{ig} - \mathbb{E}[\alpha_{ig}]) + (\gamma_{ig} - \mathbb{E}[\gamma_{ig}]) \bar{D}_{ig}}_{\varepsilon_{ig}}
\end{aligned}
$$

IV Estimand

$$
\gamma_{IV} = \frac{\text{Cov}(Y_{ig}, S_g)}{\text{Cov}(\bar{D}_{ig}, S_g)} = \gamma + \frac{\text{Cov}(\varepsilon_{ig}, S_g)}{\text{Cov}(\bar{D}_{ig}, S_g)} = \ldots = \gamma + \frac{\text{Cov}(\gamma_{ig}, \bar{C}_{ig})}{\mathbb{E}(\bar{C}_{ig})}
$$

Identification – Average Spillover Effect when Untreated

One-sided Noncompliance

$$(1 - Z_{ig})Y_{ig} = (1 - Z_{ig})(\alpha_{ig} + \cancel{\beta_{ig} D_{ig}} + \gamma_{ig}\bar{D}_{ig} + \cancel{\delta_{ig} D_{ig}\bar{D}_{ig}}) = (1 - Z_{ig}) \begin{pmatrix} 1 \\ \bar{D}_{ig} \end{pmatrix}' \begin{pmatrix} \alpha_{ig} \\ \gamma_{ig} \end{pmatrix}$$

Theorem

$(Z_{ig}, \bar{D}_{ig}) \perp\!\!\!\perp (\alpha_{ig}, \gamma_{ig}) | (\bar{C}_{ig}, N_g)$.

$$\begin{aligned}
\mathbb{E}\left[\left. \begin{pmatrix} 1 \\ \bar{D}_{ig} \end{pmatrix} (1 - Z_{ig})Y_{ig} \right| \bar{C}_{ig}, N_g \right] &= \mathbb{E}\left[\left. (1 - Z_{ig}) \begin{pmatrix} 1 & \bar{D}_{ig} \\ \bar{D}_{ig} & \bar{D}_{ig}^2 \end{pmatrix} \begin{pmatrix} \alpha_{ig} \\ \gamma_{ig} \end{pmatrix} \right| \bar{C}_{ig}, N_g \right] \\
&= \underbrace{\mathbb{E}\left[\left. (1 - Z_{ig}) \begin{pmatrix} 1 & \bar{D}_{ig} \\ \bar{D}_{ig} & \bar{D}_{ig}^2 \end{pmatrix} \right| \bar{C}_{ig}, N_g \right]}_{\equiv \mathbf{Q}_0(\bar{C}_{ig}, N_g)} \mathbb{E}\left[\left. \begin{pmatrix} \alpha_{ig} \\ \gamma_{ig} \end{pmatrix} \right| \bar{C}_{ig}, N_g \right]
\end{aligned}$$

Average Spillover, Untreated: $\mathbb{E}[Y_{ig}(0, \bar{d})] = \mathbb{E}(\alpha_{ig}) + \mathbb{E}(\gamma_{ig})\bar{d}$

$$\begin{bmatrix} \mathbb{E}(\alpha_{ig}) \\ \mathbb{E}(\gamma_{ig}) \end{bmatrix} = \mathbb{E}\left[\mathbf{Q}_0(\bar{C}_{ig}, N_g)^{-1} \begin{pmatrix} 1 \\ \bar{D}_{ig} \end{pmatrix} (1 - Z_{ig}) Y_{ig}\right]$$

$$\mathbf{Q}_0(\bar{C}_{ig}, N_g) \equiv \mathbb{E}\left[(1 - Z_{ig}) \begin{pmatrix} 1 & \bar{D}_{ig} \\ \bar{D}_{ig} & \bar{D}_{ig}^2 \end{pmatrix} \middle| \bar{C}_{ig}, N_g\right]$$

$\mathbf{Q}_0$ is a *known function*

Distribution of $\bar{D}_{ig}|(\bar{C}_{ig}, N_g)$ determined by experimental design.

Rank Condition: $Y_{ig}(D_{ig}, \bar{D}_{ig}) = \mathbf{f}(\bar{D}_{ig})' \left[(1 - D_{ig})\,\boldsymbol{\theta}_{ig} + D_{ig}\boldsymbol{\psi}_{ig}\right]$

$$\boxed{\mathbf{Q}_z(\bar{c}, n) \equiv \mathbb{E}\left[\mathbb{1}(Z_{ig} = z)\mathbf{f}(\bar{D}_{ig})\mathbf{f}(\bar{D}_{ig})' \,\middle|\, \bar{C}_{ig} = \bar{c}, N_g = n\right], \quad z = 0, 1}$$

Rank Condition

$\mathbf{Q}_0(\bar{c}, n), \mathbf{Q}_1(\bar{c}, n)$ invertible for all $(\bar{c}, n)$ in the support of $(\bar{C}_{ig}, N_g)$.

E.g. Linear Model

$$\mathbf{Q}_0(\bar{c}, n) = \begin{bmatrix} \mathbb{E}\{1 - S_g\} & \bar{c}\,\mathbb{E}\{S_g(1 - S_g)\} \\ \bar{c}\,\mathbb{E}\{S_g(1 - S_g)\} & \bar{c}^2\,\mathbb{E}\left\{S_g^2(1 - S_g)\right\} + \frac{\bar{c}}{n-1}\mathbb{E}\left\{S_g(1 - S_g)^2\right\} \end{bmatrix}$$

$$\mathbf{Q}_1(\bar{c}, n) = \begin{bmatrix} \mathbb{E}\{S_g\} & \bar{c}\,\mathbb{E}\left\{S_g^2\right\} \\ \bar{c}\,\mathbb{E}\left\{S_g^2\right\} & \bar{c}^2\,\mathbb{E}\left\{S_g^3\right\} + \frac{\bar{c}}{n-1}\mathbb{E}\left\{S_g^2(1 - S_g)\right\} \end{bmatrix}$$

(Rank Condition) + (Assumptions i–iv) $\Rightarrow$ Point Identified Effects

Spillover

$\bar{D}_{ig} \to Y_{ig}$ for the population, holding $D_{ig} = 0$.

Direct Effect on the Treated

$D_{ig} \to Y_{ig}$ for compliers as a function of $\bar{d}$.

Indirect Effects on the Treated

$\bar{D}_{ig} \to Y_{ig}$ for compliers holding $D_{ig} = 0$ or $D_{ig} = 1$.

Indirect Effect on the Untreated

$\bar{D}_{ig} \to Y_{ig}$ for never-takers holding $D_{ig} = 0$.

Feasible Estimation: Just-Identified IV with "Generated" Instruments

$$\widehat{\boldsymbol{\vartheta}} \equiv \left(\sum_{g=1}^{G}\sum_{i=1}^{N_g} \widehat{\boldsymbol{\mathcal{Z}}}_{ig}\mathbf{X}_{ig}'\right)^{-1}\left(\sum_{g=1}^{G}\sum_{i=1}^{N_g} \widehat{\boldsymbol{\mathcal{Z}}}_{ig} Y_{ig}\right), \quad \widehat{C}_{ig} \equiv \bar{D}_{ig}/\bar{Z}_{ig}$$

Example From Above: $Y_{ig} = \alpha + \gamma \bar{D}_{ig} + \varepsilon_{ig}$

$$\widehat{\boldsymbol{\vartheta}} = \begin{bmatrix} \widehat{\alpha} \\ \widehat{\gamma} \end{bmatrix}, \quad \mathbf{X}_{ig} = \begin{bmatrix} 1 \\ \bar{D}_{ig} \end{bmatrix}, \quad \widehat{\boldsymbol{\mathcal{Z}}}_{ig} = (1 - Z_{ig})\mathbf{Q}_0(\widehat{C}_{ig}, N_g)^{-1}\begin{bmatrix} 1 \\ \bar{D}_{ig} \end{bmatrix}$$

$$\mathbf{Q}_0(\bar{c}, n) = \begin{bmatrix} \mathbb{E}\left\{1 - S_g\right\} & \bar{c}\,\mathbb{E}\left\{S_g(1 - S_g)\right\} \\ \bar{c}\,\mathbb{E}\left\{S_g(1 - S_g)\right\} & \bar{c}^2\,\mathbb{E}\left\{S_g^2(1 - S_g)\right\} + \frac{\bar{c}}{n-1}\mathbb{E}\left\{S_g(1 - S_g)^2\right\} \end{bmatrix}$$

Crepon Example: Labor Market Displacement Effects

(SEs clustered at labor market level)

$\mathbb{E}[\gamma_{ig}\|\text{Type}]$	Popn.	Never-takers	Compliers
$\mathbb{P}$(Long-term Employment)	-0.09	0.14	-0.51
	(0.07)	(0.09)	(0.24)
$\mathbb{P}$(Any Employment)	-0.11	0.14	-0.56
	(0.06)	(0.09)	(0.24)

$$\mathbb{E}[Y_{ig}(0,\bar{d})|\text{Type}] = \mathbb{E}[\alpha_{ig}|\text{Type}] + \mathbb{E}[\gamma_{ig}|\text{Type}] \times \bar{d}$$

Crepon Example: Protective Effect of Treatment for Compliers

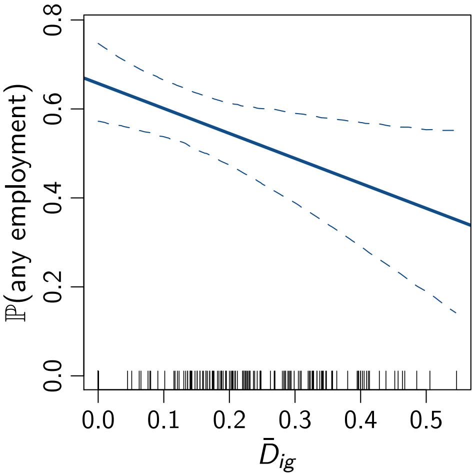
(SEs clustered at labor market level)

	α^c	γ^c	β^c	δ^c
$\mathbb{P}$(Long-term Employment)	0.48	-0.51	-0.09	0.62
	(0.04)	(0.24)	(0.05)	(0.25)
$\mathbb{P}$(Any Employment)	0.66	-0.56	-0.10	0.62
	(0.04)	(0.24)	(0.05)	(0.25)

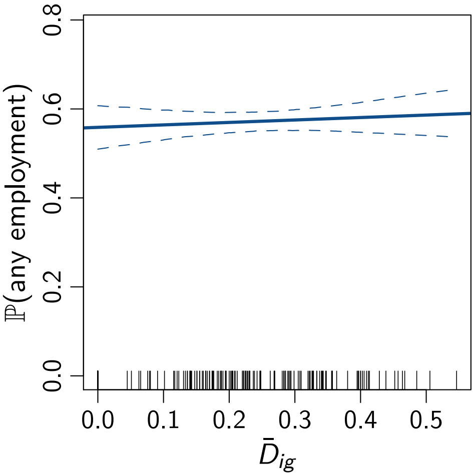
$$\mathbb{E}[Y_{ig}(0,\bar{d})|\text{Complier}] = \alpha^c + \gamma^c \cdot \bar{d}$$

$$\mathbb{E}[Y_{ig}(1,\bar{d})|\text{Complier}] = (\alpha^c + \beta^c) + (\gamma^c + \delta^c) \times \bar{d}$$

Untreated: $\mathbb{E}[Y_{ig}(0, \bar{d})|\text{Complier}]$



Treated: $\mathbb{E}[Y_{ig}(1, \bar{d})|\text{Complier}]$



Conclusion

Identification

Go beyond ITTs to identify average direct and indirect effects in randomized saturation experiments with 1-sided non-compliance.

Estimation

Simple asymptotically normal estimator under large/many-group asymptotics.

Application

Negative spillovers for those willing to take up the program offset by positive direct treatment effects: selection on gains.