Second Midterm Examination

Econ 103, Statistics for Economists

November 7, 2012

You will have 70 minutes to complete this exam. Graphing calculators, notes, and textbooks are not permitted.

I pledge that, in taking and preparing for this exam, I have abided by the University of Pennsylvania's Code of Academic Integrity. I am aware that any violations of the code will result in a failing grade for this course.

Name: ____________________

Student ID #: ____________________

Signature: ____________________

Question:	1	2	3	4	5	6	7	8	Total
Points:	20	15	20	15	20	15	15	20	140
Score:									

Instructions: Answer all questions in the space provided. Should you run out of space, continue on the back of the page. Show your work for full credit but be aware that writing down irrelevant information will not gain you points. Be sure to sign the academic integrity statement above and to write your name and student ID number on *each page* in the space provided. Make sure that you have all pages of the exam before starting.

1. Suppose that X is a random variable with support $\{1, 2\}$ and Y is a random variable with support $\{0, 1\}$ where X and Y have the following joint distribution:

$$\begin{aligned} p_{XY}(1,0) &= 0.30 \\ p_{XY}(1,1) &= 0.25 \\ p_{XY}(2,0) &= 0.20 \\ p_{XY}(2,1) &= 0.25 \end{aligned}$$

   (a) (2 points) Express the joint probability mass function (pmf) in a 2×2 table.

   (b) (3 points) Using the table, calculate the marginal pmfs of X and Y.

   (c) (5 points) Calculate the conditional pmfs of $Y|X = 1$ and $Y|X = 2$.

(d) (3 points) Calculate $\mathbb{E}[Y|X=1]$ and $\mathbb{E}[Y|X=2]$.

(e) (7 points) Calculate the covariance between X and Y.

2. The random variables X_1 and X_2 correspond to the annual returns of Stock 1 and Stock 2. Suppose that $\mathbb{E}[X_1] = 0.1$, $\mathbb{E}[X_2] = 0.3$, $Var(X_1) = Var(X_2) = 1$, and $\rho = Corr(X_1, X_2)$. A portfolio $\Pi(\omega)$ is defined by the proportion ω of Stock # 1 that it contains. That is, $\Pi(\omega) = \omega X_1 + (1-\omega)X_2$ where $0 \leq \omega \leq 1$.

   (a) (3 points) What value of ω gives a portfolio with expected return 0.15?

   (b) (6 points) Suppose that $\omega = 1/4$. In terms of ρ, what is the portfolio variance?

(c) (3 points) Again, suppose that $\omega = 1/4$. What are the maximum and minimum values of the portfolio variance? What are the corresponding values of ρ?

(d) (3 points) If we assume that variance is a reasonable measure of risk, what does your answer to part (c) suggest about the benefits of constructing a portfolio rather than holding only one stock? Explain briefly.

3. Suppose that X is a continuous random variable with probability density function

$$f(x) = \begin{cases} 2x & \text{for } 0 \leq x \leq 1 \\ 0 & \text{elsewhere} \end{cases}$$

(a) (5 points) Calculate the cumulative distribution function, $F(x_0)$, of X.

(b) (3 points) Calculate the median of X using your answer to part (a).

(c) (5 points) Calculate $\mathbb{E}[X]$.

(d) (5 points) Calculate $\mathbb{E}[X^2]$

(e) (2 points) Using your answers to (c) and (d) along with the shortcut formula for variance, calculate $Var(X)$.

4. Suppose that $X_1, X_2, \ldots, X_n \sim$ iid Bernoulli(p). Define $S_n = \sum_{i=1}^n X_i$.

(a) (3 points) Write down the pmf and support of X_1.

(b) (2 points) Calculate $\mathbb{E}[X_1]$.

(c) (2 points) Calculate $\mathbb{E}[X_1^2]$.

(d) (3 points) Calculate $Var(X_1)$ using the shortcut formula.

(e) (5 points) What kind of random variable is S_n? Write down its pmf and support.

5. Suppose that $X_1, X_2, \ldots, X_n \sim$ iid Bernoulli(p) and define $\bar{X}_n = \frac{1}{n}\sum_{i=1}^{n} X_i$.

   (a) (3 points) Calculate $\mathbb{E}[\bar{X}_n]$.

   (b) (5 points) Calculate $Var(\bar{X}_n)$.

(c) (4 points) Is $\bar{X}_n$ an unbiased estimator of p? What about $\widehat{p} = \frac{3}{4}X_1 + \frac{1}{4}X_2$?

(d) (5 points) Suppose that $n = 2$. Which estimator has a lower variance: $\widehat{p}$ or $\bar{X}_2$? Prove your answer.

(e) (3 points) Provide some intuition for your answers to parts (c) and (d).

6. Suppose we carry out a sequence of independent Bernoulli trials, each with probability of success p, and stop as soon as we get the first success.

   (a) (2 points) What is the probability that we get a success on our first trial?

   (b) (3 points) What is the probability that we get our first success on the *second* trial? (That is, what is the probability of a Failure followed by a Success?)

(c) (5 points) What is the probability that we get our first success on the nth trial?

(d) (5 points) Suppose that we define a random variable X that equals the trial number of the first success in a sequence of independent Bernoulli trials, each with probability p of success. This is the definition of a Geometric(p) random variable. What is the probability mass function $p(x) = \mathbb{P}(X = x)$ of X? What is the support of this random variable?

7. Let X and Z be independent random variables with $\mathbb{E}[X] = \mathbb{E}[Z] = 0$, $Var(X) = \sigma_X^2$, and $Var(Z) = \sigma_Z^2$. Define $Y = aX + Z$ where a is a constant.

   (a) (2 points) What is $\mathbb{E}[Y]$?

   (b) (3 points) What is $Var(Y)$?

   (c) (5 points) What is $Cov(X, Z)$?

(d) (5 points) What is $Cov(X, Y)$?

8. Let X, Y and Z be independent normal random variables where $\mathbb{E}[X] = \mathbb{E}[Z] = 0$, $\mathbb{E}[Y] = 3$, $Var(X) = 9$, $Var(Y) = 4$ and $Var(Z) = 1$. For each of the following, unless I specifically ask you to provide an R command, please give a *numeric* answer.

   (a) (5 points) What is the approximate value of $\mathbb{P}(-1 \leq X/3 \leq 1)$?

   (b) (5 points) What value of k ensures that $\mathbb{P}(3 - k \leq Y \leq 3 + k) \approx 0.95$?

   (c) (5 points) Suppose I want to find the value of d such that $\mathbb{P}(-d \leq Z \leq d) = 0.5$. What R command should I use?

   (d) (5 points) What R command should I use to calculate the probability that the random variable Z^2 is greater than or equal to 5?