A Framework for Eliciting, Incorporating and Disciplining Identification Beliefs in Linear Models

Francis J. DiTraglia [1] Camilo García-Jimeno[2]

[1]University of Pennsylvania

[2]University of Pennsylvania and NBER

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Motivation

Workhorse Linear Model

Estimate causal effect of endogenous treatment using OLS, IV

Since 2000 in *AER Alone*

$>$ 30 papers whose main results are a comparison of OLS and IV

Three Key Challenges

Measurement Error, Endogenous Regressor, Invalid Instrument

Identifying causal effect *requires* imposing beliefs.

Two Kinds of Beliefs

Formal – Used in Estimation

E.g. assume instrument satisfies exclusion restriction

Informal – Not Used in Estimation

E.g. sign of regressor endogeneity, presence of measurement error

Problems with Informal Beliefs

Used for interpretation, e.g. reconcile OLS and IV, but:

1. "Leaving money on the table"
2. Contradictory Beliefs

Judgement Under Uncertainty: Heuristics and Biases

(Kahneman & Tversky; *Science*, 1974)

> *. . . an internally consistent set of subjective probabilities can be incompatible with other beliefs held by the individual . . . For judged probabilities to be considered adequate, or rational, internal consistency is not enough. The judgements must be compatible with the entire web of beliefs held by the individual.*

This Project

- Identified set with endog. treatment, invalid instrument & measurement error:
  1. Express using intelligible quantities
  2. Show how the three problems *interact*
- Combine with *all* researcher beliefs using Bayesian tools:
  1. Sensitivity analysis
  2. Ensure beliefs are mutually consistent
  3. Learn something that is implied by your beliefs, given data
- Two Cases:
  1. Continuous treatment, classical measurement error
  2. Binary treatment & instrument, "non-differential" meas. error

Continuous T^*, Classical Measurement Error

$$
\begin{aligned}
y &= \beta T^* + u \\
T^* &= \pi z + v \\
T &= T^* + w
\end{aligned}
$$

- Classical Measurement Error: $w \perp (T^*, z, v, u)$
- Endogenous Regressor: $Cov(T^*, u) \neq 0$
- Endogenous Instrument: $Cov(z, u) \neq 0$

The Colonial Origins of Comparative Development

Acemoglu, Johnson & Robinson (2001, AER)

$y = \log(\text{GDPc}),\ T = \text{institutions},\ z = \log(\text{settler mortality})$

$$\boxed{\text{OLS: } \underset{(0.06)}{0.52} \qquad \text{IV: } \underset{(0.16)}{0.94}}$$

Researcher Beliefs

- Settler mortality is exogenous, i.e. $Cov(z, u) = 0$
- $Cov(T^*, u) > 0$ & classical measurement error in institutions

"[The IV estimate is] in fact larger than the OLS estimates reported in Table 2. This suggests that measurement error in the institutions variables that creates attenuation bias is likely to be more important than reverse causality and omitted variables biases."

Parameters: Interpretable, Scale-Free, Bounded Support

Reduced Form (Identified)

Correlations between observables: $\rho_{Ty}, \rho_{Tz}, \rho_{zy}$

Structural (Un-identified)

1. Regressor Endogeneity: ρ_{T^*u}
2. Instrument Endogeneity: ρ_{zu}
3. Classical Measurement Error: κ

$$\kappa \equiv \frac{Var(T^*)}{Var(T)} \in (0, 1]$$

Identified Set for $(\rho_{T^*u}, \rho_{uz}, \kappa)$

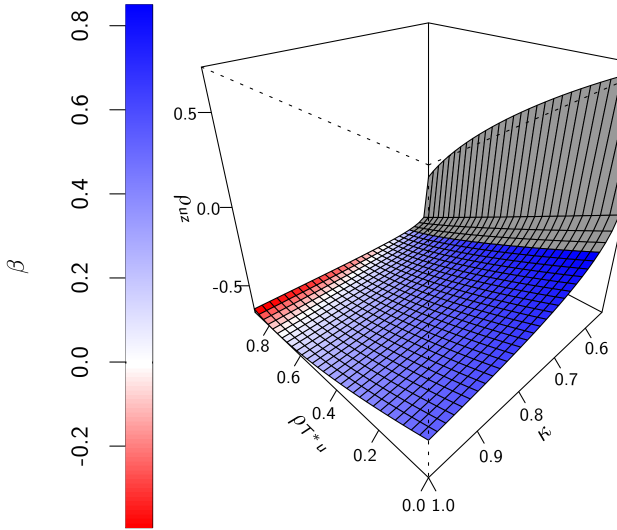
$$\rho_{uz} = \left(\frac{\rho_{T^*u}\rho_{Tz}}{\sqrt{\kappa}}\right) - (\rho_{Ty}\rho_{Tz} - \kappa\rho_{zy})\sqrt{\frac{1-\rho_{T^*u}^2}{\kappa\left(\kappa - \rho_{Ty}^2\right)}}$$

$$\kappa \in \left(\frac{\rho_{Ty}^2 + \rho_{Tz}^2 - 2\rho_{Ty}\rho_{Tz}\rho_{zy}}{1-\rho_{zy}^2}, 1\right]$$

$$\rho_{T^*u} \in (-1, 1)$$

- Any two of $(\rho_{T^*u}, \rho_{uz}, \kappa)$ determine the third, given data.
- $\kappa \in (\underline{\kappa}, 1] \iff$ upper bound for extent of measurement error.
- Data alone restrict neither treatment endogeneity ρ_{T^*u} nor β.
- One-sided bound for ρ_{uz}, but never rules out $\rho_{uz} = 0$.

Colonial Origins: Identified Set at MLE for $\rho_{T^*u} > 0$



Transparent Parameterization

Reduced Form Parameters

$\varphi = (\rho_{Ty}, \rho_{Tz}, \rho_{zy})$

Structural Parameters

$\theta = (\rho_{T^*u}, \rho_{zu}, \kappa)$

Bayesian Inference

- Inference for φ is standard.
- Identified set $\Theta(\varphi)$ for θ updated by data only through φ.
- Beliefs enter by placing conditional prior on Θ given φ.

Bayesian Inference

Step I: Reduced Form

Draw $\varphi^{(j)}$ from posterior for $\varphi = (\rho_{Ty}, \rho_{Tz}, \rho_{zy})$.

Step II: Structural

Option A – "Frequentist Friendly"

1. Place sign/interval restrictions $\mathcal{R}$ on $\Theta(\varphi^{(j)})$
2. Bounds for $\theta = (\rho_{T^*u}, \rho_{uz}, \kappa)$ and β given $\varphi^{(j)}$
3. Inference for identified set

Option B – "Fully Bayesian"

1. Uniform prior on $\Theta(\varphi^{(j)}) \cap \mathcal{R}$
2. Draw $\theta^{(j)}$ given $\varphi^{(j)}$ and calculate implied $\beta^{(j)}$
3. Inference for parameters

Colonial Origins: Inference Under $\rho_{T^*u} \in [0, 0.9]$

Under $\kappa < 0.6$

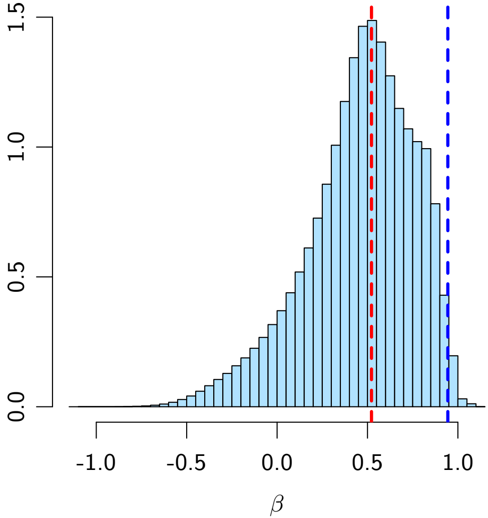
Empty identified set with approximately 30% probability!

Under $\kappa > 0.6$

Frequentist-Friendly			Fully Bayesian	
P(Valid)	$\underline{\beta}$	$\bar{\beta}$	ρ_{uz}	β
0.27	-0.47	0.85	-0.57	0.49
	$[-0.71, -0.25]$	$[0.70, 1.00]$	$[-0.81, -0.15]$	$[0.00, 0.93]$

Implications for β When $\rho_{T^*u} > 0$ and $\kappa \in (0.6, 1]$

Red = OLS Estimate, Blue = IV Estimate



Binary Treatment and Instrument

$$y = c + \beta T^* + u$$

Endogenous Treatment

$$\delta_{T^*} = \mathbb{E}[u|T^* = 1] - \mathbb{E}[u|T^* = 0]$$

Invalid Instrument

$$\delta_z = \mathbb{E}[u|z = 1] - \mathbb{E}[u|z = 0]$$

Non-differential Measurement Error

Observe T such that

- $\alpha_0 = \mathbb{P}(T = 1|T^* = 0)$
- $\alpha_1 = \mathbb{P}(T = 0|T^* = 1)$
- $T \perp (z, y)|T^*$

Bringing Education to Afghan Girls

Burde & Linden (2013, AEJ Applied)

RCT: Build schools in treatment villages, compare girls' test scores.

- y – Girl's test score (standardized)
- T^* – Girl's true school attendance (unobserved)
- T – Parent's report of child's school attendance
- z – School built in village (randomized)
- $\mathbf{x}$ – Child and household characteristics

Afghan Girls RCT

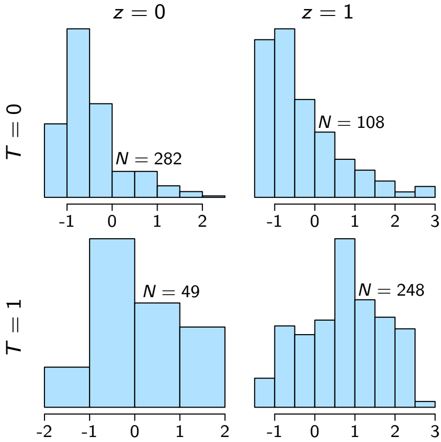
ITT	OLS	IV
0.66	0.86	1.3
(0.06)	(0.06)	(0.12)

Beliefs

- Positive selection
- $p \approx p^*$

Invalid Instrument?

- Spillovers $(+)$
- School Substitution $(-)$



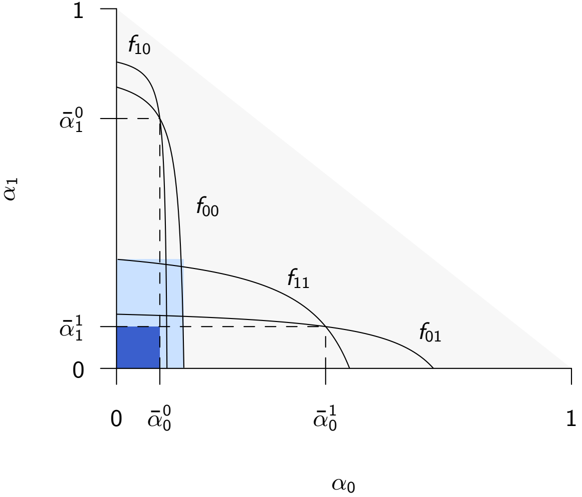
Identified Set for $(\alpha_0, \alpha_1, \delta_{T^*}, \delta_z)$

$$
\begin{aligned}
\delta_z &= B(\alpha_0, \alpha_1) + \left(\frac{p_1 - p_0}{1 - \alpha_0 - \alpha_1}\right)\delta_{T^*} \\
\alpha_0 &\in [0, \bar{\alpha}_0) \\
\alpha_1 &\in [0, \bar{\alpha}_1) \\
\delta_{T^*} &\in (-\infty, \infty)
\end{aligned}
$$

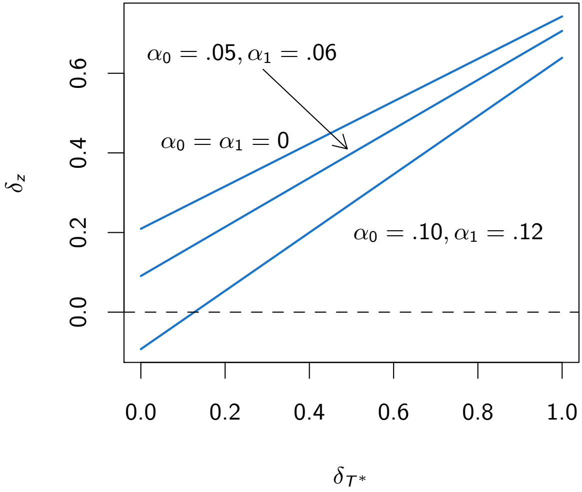
- Linear relationship between δ_{T^*} and δ_z given α_0, α_1
- $(\alpha_0 + \alpha_1) < 1 \implies \alpha_0 < \min_k\{p_k\}$, $\alpha_1 < \min_k\{1 - p_k\}$
- Tighter bounds for (α_0, α_1) from $\text{Var}(u|T^*, z) > 0$
- Data alone place no restrictions on δ_{T^*}, δ_z, or β.

Afghan Girls RCT: $\alpha_0 \leq 0.10, \alpha_1 \leq 0.12$

$\min_k p_k = 0.15, \ \min_k(1 - p_k) = 0.3$



Afghan Girls RCT: Contours of Identified Set at MLE



Yet Another Transparent Parameterization

Reduced Form Parameters

$\varphi = (\mathbb{E}[y|T,z], Var[y|T,z], \mathbb{P}[T,z])$

Structural Parameters

$\theta = (\alpha_0, \alpha_1, \delta_{T^*}, \delta_z)$

Inference

Same basic idea as in classical measurement error case.

Afghan Girls RCT: Posterior Inference

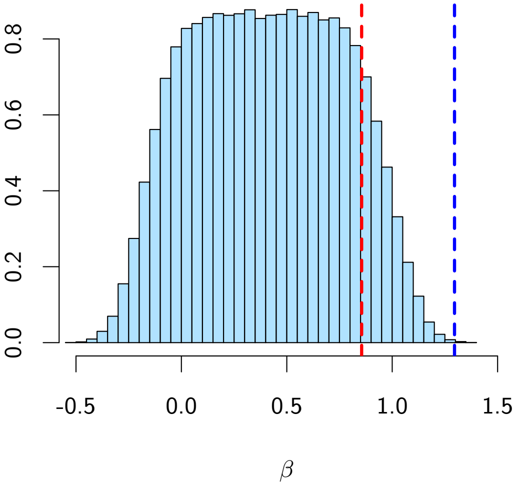
Imposing $\delta_{T^*} \in [0, 1]$, $p = p^*$

Frequentist-Friendly			
$\underline{\delta}_z$	$\bar{\delta}_z$	$\underline{\beta}$	$\bar{\beta}$
-0.04	0.73	-0.24	1.11
$[-0.11, 0.04]$	$[0.65, 0.82]$	$[-0.39, -0.08]$	$[0.96, 1.26]$

Fully Bayesian	
δ_z	β
0.40	0.41
$[0.10, 0.71]$	$[-0.15, 0.96]$

Afghan Girls RCT: Posterior for β

Red = OLS Estimate, Blue = IV Estimate



Summary

- Causal effect in linear model with measurement error, endogenous treatment & invalid instrument.
- Two cases: continuous treatment with classical measurement error; binary treatment & instrument under non-diff. measurement error.
- Incorporate, discipline and interrogate researcher beliefs.
- Implicit researcher beliefs can be very informative in practice.

Extensions and Related Work

- R and STATA packages to implement our methods
- Heterogeneous treatment effects
- What does a *valid* instrument yield in the binary-binary setting?