Lab #7 - More on Regression in R

Econ 224 September 18th, 2018

Robust Standard Errors

Your reading assignment from Chapter 3 of ISL briefly discussed two ways that the standard regression inference formulas built into R can go wrong: (1) non-constant error variance, and (2) correlation between regression errors. Today we'll briefly look at the first of these problems and how to correct for it.

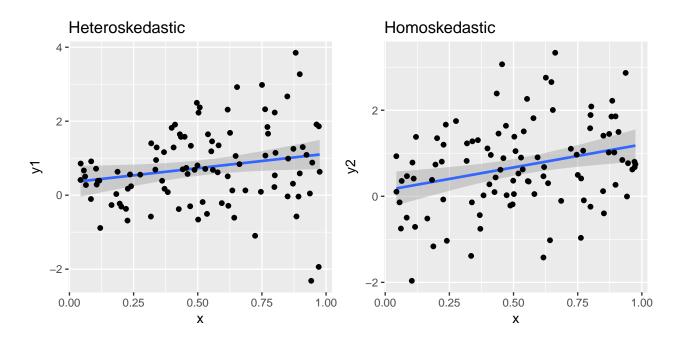
Consider the simple linear regression $y_i = \beta_0 + \beta_1 x_i + \epsilon_i$. If the variance of ϵ_i is unrelated to the value of the predictor x_i , we say that the regression errors are *homoskedastic*. This is just a fancy Greek work for *constant variance*. If instead, the variance of ϵ_i depends on the value of x_i , we say that the regression errors are *heteroskedastic*. This is just a fancy Greek word for *non-constant variance*. Heteroskedasticity does not invalidate our least squares estimates of β_0 and β_1 , but it does invalidate the formulas used by lm to calculate standard errors and p-values.

Let's look at a simple simulation example:

```
set.seed(4321)
n <- 100
x <- runif(n)
e1 <- rnorm(n, mean = 0, sd = sqrt(2 * x))
e2 <- rnorm(n, mean = 0, sd = 1)
intercept <- 0.2
slope <- 0.9
y1 <- intercept + slope * x + e1
y2 <- intercept + slope * x + e2
library(tidyverse)
mydat <- tibble(x, y1, y2)
rm(x, y1, y2)</pre>
```

From the simulation code, we see that the errors e1 are heteroskedastic since their standard deviation is a function of x. In contrast, the errors e2 are homoskedastic since their standard deviation is *not* a function of x. This means that a regression of y1 on x will exhibit heteroskedasticity but a regression of y2 on x will not. We can see this from a plot of the data:

```
library(ggplot2)
library(gridExtra)
heterosked_plot <- ggplot(mydat, aes(x, y1)) +
  geom_smooth(method = 'lm') +
  geom_point() +
  ggtitle('Heteroskedastic')
homosked_plot <- ggplot(mydat, aes(x, y2)) +
  geom_smooth(method = 'lm') +
  geom_point() +
  ggtitle('Homoskedastic')
grid.arrange(heterosked_plot, homosked_plot, ncol = 2)</pre>
```



The values of y1 "fan out" around the regression line since e1 becomes *more variable* as x increases. In contrast, the values of y2 do not show such a pattern: the variability in e2 is unrelated to x.

lm_robust

We'll use the function lm_robust in the package estimatr to calculate the appropriate standard errors for a regression with heteroskedasticity. Make sure to install this package before proceeding. For more information on estimatr, see the help files and https://declaredesign.org/r/estimatr/. Standard errors that account for heteroskedasticity are often called *robust* because they do not depend on the fairly strong assumption of constant error variance. The function lm_robust is nearly identical to lm but it allows us to specify a new argument se_type to indicate what kinds of standard errors we want to use. If we set se_type = 'classical' we'll get exactly the same standard errors as if we had used lm, namely standard errors that assume homoskedasticity:

```
library(estimatr)
reg classical <- lm robust(y1 ~ x, mydat, se type = 'classical')
summary(reg_classical)
Call:
lm_robust(formula = y1 ~ x, data = mydat, se_type = "classical")
Standard error type: classical
Coefficients:
            Estimate Std. Error t value Pr(>|t|) CI Lower CI Upper DF
                         0.2240
                                  1.526 0.13027 -0.10273
                                                            0.7863 98
(Intercept)
             0.3418
              0.7766
                         0.3785
                                  2.052 0.04286 0.02548
                                                            1.5277 98
х
Multiple R-squared: 0.04119 , Adjusted R-squared: 0.0314
F-statistic: 4.21 on 1 and 98 DF, p-value: 0.04286
```

If we set se_type = 'stata' we'll get heteroskedasticity-robust standard errors identical to those calculated by the command reg, robust in Stata. Since many economists still use Stata, this is handy for being able to replicate their results. Notice that the robust standard errors are *larger* than the classical ones. This is fairly typical in applications:

```
reg_robust <- lm_robust(y1 ~ x, mydat, se_type = 'stata')</pre>
summary(reg robust)
Call:
lm_robust(formula = y1 ~ x, data = mydat, se_type = "stata")
Standard error type: HC1
Coefficients:
            Estimate Std. Error t value Pr(>|t|) CI Lower CI Upper DF
(Intercept)
              0.3418
                         0.1739
                                  1.966 0.05215 -0.003241
                                                              0.6868 98
              0.7766
                         0.4068
                                  1.909 0.05919 -0.030707
                                                              1.5839 98
x
Multiple R-squared: 0.04119 , Adjusted R-squared: 0.0314
F-statistic: 3.644 on 1 and 98 DF, p-value: 0.05919
```

You should go back through the two preceding sets of regression results carefully and verify that the estimates are *identical* in each case. Because the standard errors are different, however, so are the test statistics and p-values. For example, x is significant at the 5% level when we use classical standard errors, but not when we use heteroskedasticity-robust standard errors. In general, failing to account for heteroskedasticity leads us to *understate* the true sampling uncertainty in our regression estimates.

F-tests with lm_robust

Heteroskedasticity doesn't just invalidate inference based on the t-tests from the lm summary output; it also invalidates any F-tests that we construct by passing these results to linearHypothesis. Fortunately, lm_robust makes it easy to fix this problem: as long as we fit our regression using lm_robust in place of lm and choose robust standard errors, when we pass the regression object to linearHypothesis, it will automatically make the appropriate adjustments. For example, notice that these *do not* give the same results:

```
library(car)
linearHypothesis(reg_classical, 'x = 0')
Linear hypothesis test
Hypothesis:
x = 0
Model 1: restricted model
Model 2: y1 ~ x
Res.Df Df Chisq Pr(>Chisq)
1 99
2 98 1 4.2098 0.04019 *
---
Signif. codes: 0 '***' 0.001 '**' 0.01 '*' 0.05 '.' 0.1 ' ' 1
```

linearHypothesis(reg_robust, 'x = 0')
Linear hypothesis test
Hypothesis:
x = 0
Model 1: restricted model
Model 2: y1 ~ x

 Res.Df Df Chisq Pr(>Chisq)
1 99
2 98 1 3.6442 0.05626 .
--Signif. codes: 0 '***' 0.001 '**' 0.01 '*' 0.05 '.' 0.1 ' ' 1

That is because the first one uses classical standard errors while the second uses heteroskedasticity-robust standard errors.

Exercise #1

- (a) Fit a regression predicting colgpa from hsize, hsize², hsperc, sat, female and athlete based on the college_gpa.csv dataset from Problem Set #3. (You can download the data from the course website.
- (b) Compare the *classical* and *robust* standard errors for each predictor in this model. Are they similar or very different?
- (c) Test the null hypothesis that hsperc, sat, female, and athlete provide no additional predictive information after controlling for hsize and hsize². Carry out the test two ways: first using *classical* and then using *robust* standard errors. How do the results differ?

Solution to Exercise #1

In this particular example, robust versus classical standard errors give very similar results:

```
gpa <- read_csv('http://ditraglia.com/econ224/college_gpa.csv')
mymodel <- colgpa ~ hsize + I(hsize^2) + hsperc + sat + female + athlete
classical <- lm_robust(mymodel, se_type = 'classical', gpa)
robust <- lm_robust(mymodel, se_type = 'stata', gpa)
SE_classical <- summary(classical)$coefficients[,2]
SE_robust <- summary(robust)$coefficients[,2]
round(cbind(coef(classical), SE_classical, SE_robust), 4)</pre>
```

		SE_classical	SE_robust
(Intercept)	1.2414	0.0795	0.0799
hsize	-0.0569	0.0164	0.0169
I(hsize^2)	0.0047	0.0022	0.0023
hsperc	-0.0132	0.0006	0.0006
sat	0.0016	0.0001	0.0001
female	0.1549	0.0180	0.0179
athlete	0.1693	0.0423	0.0370

```
myrestriction <- c('hsperc = 0', 'sat = 0', 'female = 0', 'athlete = 0')</pre>
linearHypothesis(classical, myrestriction)
Linear hypothesis test
Hypothesis:
hsperc = 0
sat = 0
female = 0
athlete = 0
Model 1: restricted model
Model 2: colgpa ~ hsize + I(hsize^2) + hsperc + sat + female + athlete
  Res.Df Df Chisq Pr(>Chisq)
   4134
1
2
   4130 4 1678.9 < 2.2e-16 ***
Signif. codes: 0 '***' 0.001 '**' 0.01 '*' 0.05 '.' 0.1 ' ' 1
linearHypothesis(robust, myrestriction)
Linear hypothesis test
Hypothesis:
hsperc = 0
sat = 0
female = 0
athlete = 0
Model 1: restricted model
Model 2: colgpa ~ hsize + I(hsize^2) + hsperc + sat + female + athlete
 Res.Df Df Chisq Pr(>Chisq)
1
   4134
   4130 4 1782.6 < 2.2e-16 ***
2
___
Signif. codes: 0 '***' 0.001 '**' 0.01 '*' 0.05 '.' 0.1 ' ' 1
```

Publication-quality Tables

A crucial part of communicating our results in a statistical analysis creating tables that are clear, and easy to read. In this section we'll look at two packages that produce publication-quality tables like those that appear in academic journals: **stargazer** and **texreg**. Make sure to install these packages before proceeding.

A Table of Summary Statistics with stargazer

We'll start by learning how to make a simple table of summary statistics. There are a few quirks to be aware of here, so **please read this paragraph carefully!** The first thing you should know is that **stargazer** can only construct summary statistics for a *dataframe*. For almost all intents and purposes a tibble *is* a

dataframe, but stargazer is an exception to this rule. If you have a tibble called, say tbl, then you will need to pass it into stargazer wrapped inside as.data.frame(). The second thing you should know is that using stargazer with knitr will not work unless you set the chunk option results = 'asis'. The third thing you need to know is that stargazer requires you to explicitly specify the kind of output that you want it to produce. If you will be knitting a pdf you'll need to set type = 'latex'. If you will be knitting an html document, you'll need to set type = 'html'. Finally, if you just want to display your table without knitting it, e.g. as a preview in RStudio, you'll need to set type = 'text'. Here is an example of the code that I ran to generate a pdf version of this document:

library(stargazer)
stargazer(mtcars, type = 'latex')

% Table created by stargazer v.5.2.2 by Marek Hlavac, Harvard University. E-mail: hlavac at fas.harvard.edu % Date and time: Sun, Sep 09, 2018 - 03:38:24 PM

Table 1:							
Statistic	Ν	Mean	St. Dev.	Min	Pctl(25)	Pctl(75)	Max
mpg	32	20.091	6.027	10	15.4	22.8	34
cyl	32	6.188	1.786	4	4	8	8
disp	32	230.722	123.939	71	120.8	326	472
hp	32	146.688	68.563	52	96.5	180	335
drat	32	3.597	0.535	2.760	3.080	3.920	4.930
wt	32	3.217	0.978	1.513	2.581	3.610	5.424
qsec	32	17.849	1.787	14.500	16.892	18.900	22.900
vs	32	0.438	0.504	0	0	1	1
am	32	0.406	0.499	0	0	1	1
gear	32	3.688	0.738	3	3	4	5
carb	32	2.812	1.615	1	2	4	8

set type to 'html' if knitting to html and 'text' if previewing in RStudio

The **stargazer** command provides dozens of options for customizing the appearance of the output it generates. Here's a nicer version of the preceding table that uses some of these options:

```
mylabels <- c('Miles/gallon',</pre>
              'No. of cylinders',
              'Displacement (cubic inches)',
              'Horsepower',
              'Rear axle ratio',
              'Weight (10001b)',
              '1/4 Mile Time',
              'V/S',
              'Manual Transmission? (1 = Yes)',
              'No. forward gears',
              'No. carburetors')
stargazer(mtcars,
          type = 'latex',
          title = 'Summary Statistics: Motor Trend Cars Dataset',
          digits = 1,
          header = FALSE,
          covariate.labels = mylabels)
```

Statistic	Ν	Mean	St. Dev.	Min	Pctl(25)	Pctl(75)	Max
Miles/gallon	32	20.1	6.0	10	15.4	22.8	34
No. of cylinders	32	6.2	1.8	4	4	8	8
Displacement (cubic inches)	32	230.7	123.9	71	120.8	326	472
Horsepower	32	146.7	68.6	52	96.5	180	335
Rear axle ratio	32	3.6	0.5	2.8	3.1	3.9	4.9
Weight (1000lb)	32	3.2	1.0	1.5	2.6	3.6	5.4
1/4 Mile Time	32	17.8	1.8	14.5	16.9	18.9	22.9
V/S	32	0.4	0.5	0	0	1	1
Manual Transmission? $(1 = Yes)$	32	0.4	0.5	0	0	1	1
No. forward gears	32	3.7	0.7	3	3	4	5
No. carburetors	32	2.8	1.6	1	2	4	8

Table 2: Summary Statistics: Motor Trend Cars Dataset

set type to 'html' if knitting to html and 'text' if previewing in RStudio

Notice how I reduced the number of significant figures presented in the table, added a caption and meaningful variable names. We can also customize which summary statistics are reported using the options summary.stat and omit.summary.stat. For example, if we only wanted to show the mean, standard deviation, and quartiles of the data, we could use the following:

Statistic	Mean	St. Dev.	Pctl(25)	Median	Pctl(75)
Miles/gallon	20.1	6.0	15.4	19.2	22.8
No. of cylinders	6.2	1.8	4	6	8
Displacement (cubic inches)	230.7	123.9	120.8	196.3	326
Horsepower	146.7	68.6	96.5	123	180
Rear axle ratio	3.6	0.5	3.1	3.7	3.9
Weight (1000lb)	3.2	1.0	2.6	3.3	3.6
1/4 Mile Time	17.8	1.8	16.9	17.7	18.9
V/S	0.4	0.5	0	0	1
Manual Transmission? $(1 = Yes)$	0.4	0.5	0	0	1
No. forward gears	3.7	0.7	3	4	4
No. carburetors	2.8	1.6	2	2	4

Table 3: Summary Statistics: Motor Trend Cars Dataset

set type to 'html' if knitting to html and 'text' if previewing in RStudio

Exercise #2

Use stargazer to make a table of summary statistics for the college_gpa.csv dataset from Problem Set #3. Add a title, use an appropriate number of digits, and provide meaningful labels for the variables.

Solution to Exercise #2

```
gpalabels <- c('Combined SAT Score',</pre>
               'Total hours through Fall',
               'College GPA (out of 4.0)',
               'Athlete? (1 = Yes)',
               'Ratio of SAT Verbal/Math',
               'Size HS Grad. Class (100s)',
               'Rank in HS Grad. Class',
               'Percentile in HS Grad. Class',
               'Female? (1 = Yes)',
               'White? (1 = Yes)',
               'Black? (1 = Yes)')
stargazer(as.data.frame(gpa), type = 'latex',
          header = FALSE,
          title = 'Summary Statistics: College GPA Dataset',
          digits = 1,
          summary.stat = c('mean', 'sd', 'p25', 'median', 'p75'),
          covariate.labels = gpalabels)
```

Statistic	Mean	St. Dev.	Pctl(25)	Median	Pctl(75)
Combined SAT Score	1,030.3	139.4	940	1,030	1,120
Total hours through Fall	52.8	35.3	17	47	80
College GPA (out of 4.0)	2.7	0.7	2.2	2.7	3.1
Athlete? $(1 = Yes)$	0.05	0.2	0	0	0
Ratio of SAT Verbal/Math	0.9	0.1	0.8	0.9	1.0
Size HS Grad. Class (100s)	2.8	1.7	1.6	2.5	3.7
Rank in HS Grad. Class	52.8	64.7	11	30	70
Percentile in HS Grad. Class	19.2	16.6	6.4	14.6	27.7
Female? $(1 = Yes)$	0.4	0.5	0	0	1
White? $(1 = \text{Yes})$	0.9	0.3	1	1	1
Black? $(1 = Yes)$	0.1	0.2	0	0	0

Table 4: Summary Statistics: College GPA Dataset

set type to 'html' if knitting to html and 'text' if previewing in RStudio

Regression Output with stargazer

As we have seen, when you pass a dataframe to **stargazer**, its default is to construct a table of summary statistics. If you instead pass a *regression* object, it will make a regression table. For example: Run a bunch of regressions using **mtcars**

	Dependent variable:
	mpg
disp	-0.04^{***}
	(0.005)
Constant	29.6***
	(1.2)
Observations	32
\mathbb{R}^2	0.7
Adjusted \mathbb{R}^2	0.7
Residual Std. Error	3.3 (df = 30)
F Statistic	76.5^{***} (df = 1; 30)
Note:	*p<0.1; **p<0.05; ***p<0.01

Table 5: Predicting Fuel Economy from Displacement

```
# set type to 'html' if knitting to html and 'text' if previewing in RStudio
```

Let's run a few more regressions and make a table that summarizes the results of *all* of them:

set type to 'html' if knitting to html and 'text' if previewing in RStudio

Notice how I added a label for the dependent variable and appended a *note* to the regression table.

		Dependent variable:		
		Miles/gallon		
	(1)	(2)	(3)	
Displacement (cubic inches)	-0.04^{***} (0.005)		-0.02^{*} (0.01)	
Weight (1000lb)		-5.3^{***} (0.6)	-3.4^{***} (1.2)	
Constant	29.6^{***} (1.2)	37.3^{***} (1.9)	35.0^{***} (2.2)	
	32 0.7	32 0.8	32 0.8	
Adjusted R ² Residual Std. Error F Statistic	$\begin{array}{c} 0.7\\ 3.3 \ (\mathrm{df}=30)\\ 76.5^{***} \ (\mathrm{df}=1;30) \end{array}$	$\begin{array}{c} 0.7\\ 3.0 \; (\mathrm{df}=30)\\ 91.4^{***} \; (\mathrm{df}=1;30) \end{array}$	$\begin{array}{c} 0.8\\ 2.9 \; (\mathrm{df}=29)\\ 51.7^{***} \; (\mathrm{df}=2;29) \end{array}$	
Note:	*p<0.1; **p<0.05; ***p<0.01 Data are courtesy of Motor Trend Magazine. Also, R rules!			

Table 6: Regression Results for Motor Trend Dataset

Exercise #3

- (a) Use 1m to create an object called reg1 that predicts colgpa from hsize and hsize^2.
- (b) Use lm to create an object called reg2 that adds hsperc, sat, female and athlete to reg1.
- (c) Use stargazer to make a summary table that compares the output of reg1 and reg2. Be sure to add a title, use appropriate labels, a reasonable number of digits, etc.

Solution to Exercise #3

% Table created by stargazer v.5.2.2 by Marek Hlavac, Harvard University. E-mail: hlavac at fas.harvard.edu % Date and time: Sun, Sep 09, 2018 - 03:38:28 PM

	Dependent variable:				
	Colleg	ge GPA			
	(1)	(2)			
Size HS Grad. Class (100s)	0.063^{***}	-0.057^{***}			
	(0.019)	(0.016)			
Size HS Grad. Class Squared	-0.011^{***}	0.005^{**}			
	(0.003)	(0.002)			
Percentile in HS Grad. Class		-0.013^{***}			
		(0.001)			
Combined SAT Score		0.002***			
		(0.0001)			
Female? $(1 = \text{Yes})$		0.155^{***}			
		(0.018)			
Athlete? $(1 = \text{Yes})$		0.169^{***}			
		(0.042)			
Constant	2.592***	1.241***			
	(0.029)	(0.079)			
Observations	4,137	4,137			
\mathbb{R}^2	0.005	0.293			
Adjusted \mathbb{R}^2	0.004	0.291			
Residual Std. Error	$0.657 \ (df = 4134)$	$0.554 \ (df = 4130)$			
F Statistic	10.187^{***} (df = 2; 4134)	$284.589^{***} (df = 6; 4130)$			
Note:	*p<0.1; **p<0.05; ***p<0.01				

 Table 7: Predicting College GPA

	Model 1	Model 2	Model 3
Intercept	29.60***	37.29***	34.96^{***}
	(1.48)	(2.19)	(2.37)
Displacement (cubic inches)	-0.04^{***}		-0.02^{*}
	(0.01)		(0.01)
Weight (1000lb)		-5.34^{***}	-3.35^{**}
		(0.65)	(1.10)
\mathbb{R}^2	0.72	0.75	0.78
Adj. \mathbb{R}^2	0.71	0.74	0.77
Num. obs.	32	32	32
RMSE	3.25	3.05	2.92

Robust Standard Errors

 Table 8: Predicting Fuel Economy

set type to 'html' if knitting to html and 'text' if previewing in RStudio

Regression Output with texreg

One downside of stargazer is that it does not play nicely with lm_robust. While there is a way to "trick" stargazer into doing the right thing with an object created by lm_robust (See https://declaredesign.org/r/ estimatr/articles/regression-tables.html for details), this is a bit of a pain. Instead we'll use an alternative to stargazer called texreg. As with stargazer you need to set the chunk option results = 'asis' to get texreg to display correctly with knitr.

The texreg package provides three main functions: texreg() is for pdf output with LaTeX, htmlreg() is for html output, and screenreg() is for text output. This is different form stargazer which has a *single* function but requires the user to specify type to indicate the desired output:

use htmlreg() if knitting to html, and screenreg() if previewing in RStudio

The output is very similar to **stargazer**. Note however that we need to pack multiple sets of regression results into a **list** object for use with **texreg**.

Exercise #4

Repeat Exercise #3 but use lm_robust to generate robust standard errors and texreg rather than stargazer to make the table of results.

	16 1 1 1	1110
	Model 1	Model 2
Intercept	2.59^{***}	1.24^{***}
	(0.03)	(0.08)
Size HS Grad. Class (100s)	0.06**	-0.06^{***}
	(0.02)	(0.02)
Size HS Grad. Class Squared	-0.01^{***}	0.00^{*}
	(0.00)	(0.00)
Rank in HS Grad Class		-0.01^{***}
		(0.00)
Combined SAT Score		0.00^{***}
		(0.00)
Female? $(1 = Yes)$		0.15^{***}
		(0.02)
Athlete? $(1 = Yes)$		0.17^{***}
		(0.04)
\mathbb{R}^2	0.00	0.29
Adj. \mathbb{R}^2	0.00	0.29
Num. obs.	4137	4137
RMSE	0.66	0.55

Note: robust standard errors.

Table 9: Predicting College GPA

Solution to Exercise #4

```
reglabels <- c('Intercept',</pre>
                'Size HS Grad. Class (100s)',
                'Size HS Grad. Class Squared',
                'Rank in HS Grad Class',
                'Combined SAT Score',
                'Female? (1 = Yes)',
                'Athlete? (1 = Yes)')
reg1_robust <- lm_robust(colgpa ~ hsize + I(hsize^2),</pre>
                          se_type = 'stata', gpa)
reg2_robust <- lm_robust(colgpa ~ hsize + I(hsize^2) + hsperc + sat + female + athlete,</pre>
                          se_type = 'stata', gpa)
texreg(list(reg1_robust, reg2_robust),
       custom.coef.names = reglabels,
       include.ci = FALSE,
       caption = 'Predicting College GPA',
       custom.note = 'Note: robust standard errors.')
```

use htmlreg() if knitting to html, and screenreg() if previewing in RStudio

Important Note:

From now own, we will expect you to format your results in problem sets and labs using the stargazer and/or texreg packages.