

# Problem Set #6

Econ 103

## Part I – Problems from the Textbook

Chapter 4: 19, 21, 23 (*When necessary, use  $R$  rather than the Normal tables in the front of the textbook.*)

## Part II – Additional Problems

1. Suppose that  $X$  is a random variable with the following PDF

$$f(x) = \begin{cases} x & 0 \leq x \leq 1 \\ 2 - x & 1 \leq x \leq 2 \\ 0 & \text{otherwise} \end{cases}$$

- (a) Graph the PDF of  $X$ .
  - (b) Show that  $\int_{-\infty}^{\infty} f(x) dx = 1$ .
  - (c) What is  $P(0.5 < X < 1.5)$ ?
2. A random variable is said to follow a Uniform( $a, b$ ) distribution if it is equally likely to take on any value in the range  $[a, b]$  and never takes a value outside this range. Suppose that  $X$  is such a random variable, i.e.  $X \sim \text{Uniform}(a, b)$ .
    - (a) What is the support of  $X$ ?
    - (b) Explain why the PDF of  $X$  is  $f(x) = 1/(b - a)$  for  $a \leq x \leq b$ , zero elsewhere.
    - (c) Using the PDF from part (b), calculate the CDF of  $X$ .
    - (d) Verify that  $f(x) = F'(x)$  for the present example.
    - (e) Calculate  $E[X]$ .
    - (f) Calculate  $E[X^2]$ . *Hint:* recall that  $b^3 - a^3$  can be factorized as  $(b - a)(b^2 + a^2 + ab)$ .
    - (g) Using the shortcut formula and parts (e) and (f), show that  $\text{Var}(X) = (b - a)^2/12$ .

3. Suppose that  $X \sim N(0, 16)$  independent of  $Y \sim N(2, 4)$ . Recall that our convention is to express the normal distribution in terms of its mean and variance, i.e.  $N(\mu, \sigma^2)$ . Hence,  $X$  has a mean of zero and variance of 16, while  $Y$  has a mean of 2 and a variance of 4. In completing some parts of this question you will need to use the R function `pnorm` described in class. In this case, please write down the command you used as well as the numeric result.
- Calculate  $P(-8 \leq X \leq 8)$ .
  - Calculate  $P(0 \leq Y \leq 4)$ .
  - Calculate  $P(-1 \leq Y \leq 6)$ .
  - Calculate  $P(X \geq 10)$ .

**Note:** In the following five questions  $X_1, X_2 \sim iid N(\mu, \sigma^2)$ ,  $Y = (X_1 - \mu)/\sigma$ ,  $Z = (X_2 - \mu)/\sigma$ .

- What is the distribution of  $X_1 + X_2$ ?
  - Use R to calculate  $P(X_1 + X_2 > 5)$  if  $\mu = 5$  and  $\sigma^2 = 50$ .
  - Use R to calculate the 10th percentile of the distribution of  $X_1 + X_2$ .
- What is the distribution of  $Y^2$ ?
  - Use R to calculate  $P(Y^2 \geq 1)$ .
- What is the distribution of  $Y^2 + Z^2$ ?
  - Use R to calculate the 95th percentile of the distribution of  $Y^2 + Z^2$ .
- What is the distribution of  $Z/\sqrt{Y^2}$ ?
  - What value of  $c$  satisfies  $P(-c \leq Z/\sqrt{Y^2} \leq c) = 0.95$ ?
  - How does the interval in part (b) compare to the corresponding interval for  $Z$ ?
- What is the distribution of  $Y^2/Z^2$ ?
  - Use R to calculate the 95th percentile of the distribution of  $Y^2/Z^2$ .