

Problem Set #4

Econ 103

Part I – Problems from the Textbook

Chapter 3: 1, 3, 5, 9, 11, 13, 15, 17, 19, 21, 23, 25, 27, 29

Part II – Additional Problems

1. Suppose you flip a fair coin twice.
 - (a) List all the basic outcomes in the sample space.
 - (b) Let A be the event that you get at least one head. List all the basic outcomes in A .
 - (c) What is the probability of A ?
 - (d) List all the basic outcomes in A^c .
 - (e) What is the probability of A^c ?
2. Suppose I deal two cards at random from a well-shuffled deck of 52 playing cards. What is the probability that I get a pair of aces?
3. Suppose everyone in a class of one hundred students flips a fair coin five times.
 - (a) What is the probability that John Smith, a particular student in the class, gets five heads in a row?
 - (b) What is the probability that at least one person gets five heads in a row?
4. (Adapted from Mosteller, 1965) A jury has three members: the first flips a coin for each decision, and each of the remaining two independently has probability p of reaching the correct decision. Call these two the “serious” jurors and the other the “flippant” juror (pun intended).
 - (a) What is the probability that the serious jurors both reach the same decision?
 - (b) What is the probability that the serious jurors each reach different decisions?
 - (c) What is the probability that the jury reaches the correct decision? Majority rules.

5. This question refers to the prediction market example from lecture. Imagine it is October 2012. Let O be a contract paying \$10 if Obama wins the election, zero otherwise, and R be a contract paying \$10 if Romney wins the election, zero otherwise. Let $\text{Price}(O)$ and $\text{Price}(R)$ be the respective prices of these contracts.
- Suppose you *buy* one of each contract. What is your profit?
 - Suppose you *sell* one of each contract. What is your profit?
 - What must be true about $\text{Price}(O)$ and $\text{Price}(R)$, to prevent an opportunity for statistical arbitrage?
 - How is your answer to part (c) related to the Complement Rule?
 - What is the implicit assumption needed for your answers to parts (a)–(c) to be correct? How would your answers change if we were to relax this assumption?
6. “Odd Question” # 6, from Hacking (2001):

You are a physician. You think it is quite likely that one of your patients has strep throat, but you aren’t sure. You take some swabs from the throat and send them to a lab for testing. The test is (like nearly all lab tests) not perfect. If the patient has strep throat, then 70% of the time the lab says yes. But 30% of the time it says NO. If the patient does not have strep throat, then 90% of the time the lab says NO. But 10% of the time it says YES. You send five successive swabs to the lab, from the same patient. and get back these results in order: YES, NO, YES, NO, YES.

Let S be the event that the patient has strep throat, and S^c be the even that she does not. Let Y be the event that a given test says YES and $N = Y^c$ be the event that a given test says NO. You may assume that the tests are independent.

- Calculate the probability that your patient has strep throat. (Hint, there is a missing piece of information and you should express your answer *in terms of it*.)
- Based on your answer to part (b) do you think the patient has strep throat? Explain.

Part III – “Challenge” Problems

These are optional. If you’re up for a challenge you’ll learn quite a bit from these.

7. (Adapted from Mosteller, 1965) “What is the least number of persons required if the probability exceeds $1/2$ that two or more of them have the same birthday? (Year of birth need not match).” [Hint: Use the Complement Rule.]

8. Formally prove the Addition Rule: $P(A \cup B) = P(A) + P(B) - P(A \cap B)$ using the basic notions of set theory we learned in class and the axioms of probability. [Hint: Try to translate the intuition from the Venn diagram into an equation.]
9. Weren't stumped by the Monte Hall problem? Try this example from Mosteller (1965):

Three prisoners, A , B , and C , with apparently equally good records have applied for parole. The parole board has decided to release two of the three, and the prisoners know this but not which two. A warder friend of prisoner A knows who are to be released. Prisoner A realizes that it would be unethical to ask the warder if he, A , is to be released, but thinks of asking for the name of *one* prisoner *other than himself* who is to be released. He thinks that before he asks, his chances of release are $2/3$. He thinks that if the warder says " B will be released," his own chances have now gone down to $1/2$, because either A and B or B and C are to be released. And so A decides not to reduce his chances by asking. However, A is mistaken in his calculations. Explain.