

| | Discrete | Continuous |
|----------------|--|--|
| Support | Countable set of values | Uncountable set of values |
| Probabilities | pmf: $P(X = x) = p(x)$ | $P(X = x) = 0 \neq f(x)$ for all x $P(a \leq X \leq b) = \int_a^b f(x)dx = F(b) - F(a)$ |
| Expected Value | $\mu_X = \mathbb{E}[X] = \sum_x xp(x)$ $\mathbb{E}[g(X)] = \sum_x g(x)p(x)$ | $\mu_X = \mathbb{E}[X] = \int_{-\infty}^{\infty} xf(x) dx$ $\mathbb{E}[g(X)] = \int_{-\infty}^{\infty} g(x)f(x) dx$ |
| Independence | $p_{XY}(x, y) = p_X(x)p_Y(y)$ | $f_{XY}(x, y) = f_X(x)f_Y(y)$ |

Table 1: Differences between Discrete and Continuous Random Variables

| Probability Mass Function $p(x)$ | Probability Density Function $f(x)$ |
|-----------------------------------|---|
| Discrete Random Variables | Continuous Random Variables |
| $p(x) = P(X = x)$ | $f(x) \neq P(X = x) = 0$ |
| $p(x) \geq 0$ | $f(x) \geq 0$ |
| $p(x) \leq 1$ | $f(x)$ can be greater than one! |
| $\sum_x p(x) = 1$ | $\int_{-\infty}^{\infty} f(x) dx = 1$ |
| $F(x_0) = \sum_{x \leq x_0} p(x)$ | $F(x_0) = \int_{-\infty}^{x_0} f(x) dx$ |

Table 2: Properties of probability mass function (pmf) versus probability density function.

| | |
|----------------------------|---|
| Definition of R.V. | $X: S \rightarrow \mathbb{R}$ (RV is a fixed function from sample space to reals) |
| Support | Set of all values the RV can take |
| CDF | $F(x_0) = P(X \leq x_0)$ |
| Definition of Variance | $\sigma_X^2 = Var(X) = E[(X - E[X])^2]$ |
| Shortcut for Variance | $Var(X) = E[X^2] - (E[X])^2$ |
| Definition of Std. Dev. | $\sigma_X = \sqrt{\sigma_X^2}$ |
| Covariance | $\sigma_{XY} = Cov(X, Y) = E[(X - E[X])(Y - E[Y])]$ |
| Cov. and Independence | X, Y indep. $\Rightarrow Cov(X, Y) = 0$ but $Cov(X, Y) = 0 \not\Rightarrow X, Y$ indep. |
| Functions and Independence | X, Y indep. $\Rightarrow g(X), h(Y)$ indep. where g, h are any functions |
| Shortcut for Covariance | $Cov(X, Y) = E[XY] - E[X]E[Y]$ |
| Definition of Correlation | $\rho_{XY} = Corr(X, Y) = \sigma_{XY} / (\sigma_X \sigma_Y)$ |
| Expectations of Functions | $E[g(X)] \neq g(E[X])$ |
| Linear Functions | <p>$E[a + bX] = a + bE[X]$ where a, b are constants and X is a RV</p> <p>$Var(a + bX) = b^2 Var(X)$ where a, b are constants and X is a RV</p> <p>$E[X_1 + \dots + X_k] = E[X_1] + \dots + E[X_k]$ where X_1, \dots, X_k are any RVs</p> <p>$Var(X_1 + \dots + X_k) = Var(X_1) + \dots + Var(X_k)$ if X_1, \dots, X_k are independent RVs</p> <p>$Var(aX + bY) = a^2 Var(X) + b^2 Var(Y) + 2abCov(X, Y)$ for any RVs X, Y and constants a, b</p> |

Table 3: Essential facts that hold for *all* random variables, continuous or discrete

| | Sample Statistic | Population Parameter | Population Parameter |
|-------------|--|---|--|
| Setup | Sample from a population | Population viewed as list of objects | Population viewed as a RV |
| Mean | $\bar{x} = \frac{1}{n} \sum_{i=1}^n x_i$ | $\mu_X = \frac{1}{N} \sum_{i=1}^N x_i$ | Discrete $\mu_X = \sum_x xp(x)$ Continuous $\mu_X = \int_{-\infty}^{\infty} xf(x) dx$ |
| Variance | $s_X^2 = \frac{1}{n-1} \sum_{i=1}^n (x_i - \bar{x})^2$ | $\sigma_X^2 = \frac{1}{N} \sum_{i=1}^N (x_i - \mu_X)^2$ | $\sigma_X^2 = E[(X - E[X])^2]$ |
| Std. Dev. | $s_X = \sqrt{s_X^2}$ | $\sigma_X = \sqrt{\sigma_X^2}$ | $\sigma_X = \sqrt{\sigma_X^2}$ |
| Covariance | $s_{XY} = \frac{1}{n-1} \sum_{i=1}^n (x_i - \bar{x})(y_i - \bar{y})$ | $\sigma_{XY} = \frac{1}{N} \sum_{i=1}^N (x_i - \mu_X)(y_i - \mu_Y)$ | $\sigma_{XY} = E[(X - \mu_X)(Y - \mu_Y)]$ |
| Correlation | $r_{XY} = s_{XY}/(s_X s_Y)$ | $\rho_{XY} = \sigma_{XY}/(\sigma_X \sigma_Y)$ | $\rho_{XY} = \sigma_{XY}/(\sigma_X \sigma_Y)$ |